

**ANNA ADARSH COLLEGE FOR WOMEN (AUTONOMOUS) CHENNAI – 600040**  
**END SEMESTER EXAMINATIONS-NOVEMBER – 2025**

**SCHEME OF VALUATION**  
**CORE X – INDUSTRY MODULE STATISTICAL METHODS**  
**SECTION – A (10 X 1 = 10 Marks)**

**Answer Any Ten Questions**

1	Out-of-control process: A process that is operating in the presence of assignable causes is said to be an out-of-control process.1	1 Mark																			
2	Total quality management (TQM) is a strategy for implementing and managing quality improvement activities on an organization wide basis.	1 Mark																			
3	Describe OC curves in acceptance sampling plans. Operating characteristic (OC) curve plots the probability of accepting the lot versus the lot fraction defective.	1 Mark																			
4	Compare single and double sampling plans. A single-sampling plan is defined by the sample size n and the acceptance number c. A double-sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced.	1 Mark																			
5	What are one – sided and two-sided variable specifications? One-sided specifications focus on a change or characteristic in a single, specific direction (e.g., "greater than" or "less than"), while two-sided specifications consider changes or characteristics in either direction (e.g., "different from" or "not equal to")	1 Mark																			
6	Outline Taguchi’s philosophy in quality improvement. Taguchi basic philosophy has three concepts: 1. Design quality into the product. 2. Achieve quality by minimizing deviation from the target. 3. Measure the cost of quality as a function of deviation from the standard.	1 Mark																			
7	Summarize the computations of ANOVA for one-way classification in a table <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Source of variation</th> <th>d.f</th> <th>Sum of squares</th> <th>Mean sum of squares</th> <th>F-ratio</th> </tr> </thead> <tbody> <tr> <td>Treatment(Ration)</td> <td>k-1</td> <td><math>S_t^2</math></td> <td><math>s_t^2 = \frac{S_t^2}{(k-1)}</math></td> <td rowspan="2"><math>F = \frac{s_t^2}{s_E^2} = F_{k-1, n-k}</math></td> </tr> <tr> <td>Error</td> <td>n-k</td> <td><math>S_E^2</math></td> <td><math>s_E^2 = \frac{S_t^2}{(n-k)}</math></td> </tr> <tr> <td>Total</td> <td>n-1</td> <td><math>S_T^2</math></td> <td></td> <td></td> </tr> </tbody> </table>	Source of variation	d.f	Sum of squares	Mean sum of squares	F-ratio	Treatment(Ration)	k-1	$S_t^2$	$s_t^2 = \frac{S_t^2}{(k-1)}$	$F = \frac{s_t^2}{s_E^2} = F_{k-1, n-k}$	Error	n-k	$S_E^2$	$s_E^2 = \frac{S_t^2}{(n-k)}$	Total	n-1	$S_T^2$			1 Mark
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Total	n-1	$S_T^2$																			
8	Write the key differences between CRD and RBD designs. In CRD the experimental units are allotted at random to the treatments, so that every unit gets the same chance of receiving every treatment. In RBD all the treatments are applied at random relatively homogeneous units within each strata or block and replicated over all the blocks.	1 Mark																			
9	Define Survival function in reliability theory. The reliability or Survival function of an item is defined as $R(t) = 1 - F(t) = \Pr(T > t) \text{ for } t > 0$	1 Mark																			

10	<p>Define the mean time to failure (MTTF) of an item.</p> <p>The mean time to failure (MTTF) of an item is defined by</p> $\text{MTTF} = E(T) = \int_0^{\infty} t f(t) dt$	1 Mark
11	<p>Write the control limits for R chart.</p> $\text{UCL} = D_4 \bar{R}$ <p>Center line = <math>\bar{R}</math></p> $\text{LCL} = D_3 \bar{R}$	1 Mark
12	<p>Define mean residual life (MRL) in reliability theory.</p> <p>The mean residual (or, remaining) life, <math>\text{MRL}(t)</math>, of the item at age <math>t</math> is</p> $\text{MRL}(t) = \mu(t) = \int_0^{\infty} R(x   t) dx = \frac{1}{R(t)} \int_t^{\infty} R(x) dx$	1 Mark

**SECTION – B (5 X 5 = 25 Marks)**  
**Answer any Five Questions**

13.	<p>Illustrate the p chart in Statistical quality control.</p> <p>Suppose that the true fraction nonconforming <math>p</math> in the production process is known or is a specified standard value. Then from equation (7.5), the center line and control limits of the fraction nonconforming control chart would be as follows:</p> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> <p style="text-align: center;"><b>Fraction Nonconforming Control Chart: Standard Given</b></p> <math display="block">\text{UCL} = p + 3 \sqrt{\frac{p(1-p)}{n}}</math> <p style="text-align: center;">Center line = <math>p</math> <span style="float: right;">(7.6)</span></p> <math display="block">\text{LCL} = p - 3 \sqrt{\frac{p(1-p)}{n}}</math> </div> <p>Depending on the values of <math>p</math> and <math>n</math>, sometimes the lower control limit <math>\text{LCL} &lt; 0</math>. In these cases, we customarily set <math>\text{LCL} = 0</math> and assume that the control chart only has an upper control limit. The actual operation of this chart would consist of taking subsequent samples of <math>n</math> units, computing the sample fraction nonconforming <math>\hat{p}</math>, and plotting the statistic <math>\hat{p}</math> on the chart. As long as <math>\hat{p}</math> remains within the control limits and the sequence of plotted points does not exhibit any systematic nonrandom pattern, we can conclude that the process is in control at the level <math>p</math>. If a point plots outside of the control limits, or if a nonrandom pattern in the plotted points is observed, we can conclude that the process fraction nonconforming has most likely shifted to a new level and the process is out of control.</p>	5 Marks
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When the process fraction nonconforming  $p$  is not known, then it must be estimated from observed data. The usual procedure is to select  $m$  preliminary samples, each of size  $n$ . As a general rule,  $m$  should be at least 20 or 25. Then if there are  $D_i$  nonconforming units in sample  $i$ , we compute the fraction nonconforming in the  $i$ th sample as

$$\hat{p}_i = \frac{D_i}{n} \quad i = 1, 2, \dots, m$$

and the average of these individual sample fractions nonconforming is

$$\bar{p} = \frac{\sum_{i=1}^m D_i}{mn} = \frac{\sum_{i=1}^m \hat{p}_i}{m} \quad (7.7)$$

The statistic  $\bar{p}$  estimates the unknown fraction nonconforming  $p$ . The center line and control limits of the control chart for fraction nonconforming are computed as follows:

**Fraction Nonconforming Control Chart: No Standard Given**

$$\begin{aligned} \text{UCL} &= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \\ \text{Center line} &= \bar{p} \\ \text{LCL} &= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \end{aligned} \quad (7.8)$$

This control chart is also often called the  $p$ -chart.

14. Explain AOQ and ATI curves in sampling plans with examples.

5 Marks

Average outgoing quality is widely used for the evaluation of a rectifying sampling plan. The average outgoing quality is the quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective  $p$ . It is simple to develop a formula for average outgoing quality (AOQ). Assume that the lot size is  $N$  and that all discovered defectives are replaced with good units. Then in lots of size  $N$ , we have 1.  $n$  items in the sample that, after inspection, contain no defectives, because all discovered defectives are replaced 2.  $N-n$  items that, if the lot is rejected, also contain no defectives 3.  $N-n$  items that, if the lot is accepted, contain  $p(N-n)$  defectives. Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to  $Pap(N-n)$ , which we may express as an average fraction defective, called the average outgoing quality,

$$\text{AOQ} = \frac{P_a p (N - n)}{N} \quad \text{AOQ} \approx P_a p$$

Average outgoing quality will vary as the fraction defective of the incoming lots varies. The curve that plots average outgoing quality against incoming lot quality is called an AOQ curve.

If the lot quality is  $0 < p < 1$ , the average amount of inspection per lot will vary between the sample size  $n$  and the lot size  $N$ . If the lot is of quality  $p$  and the probability of lot acceptance is  $P_a$ , then the average total inspection per lot will be

$$\text{ATI} = n + (1 - P_a)(N - n)$$

The curve that plots average total inspection as a function of lot quality is called an ATI curve.

For Explaining the above with example

15. Examine the advantages and disadvantages of variable sampling plan in quality control.

CO3

	<p>The primary advantage of variables sampling plans is that the same operating-characteristic curve can be obtained with a smaller sample size than would be required by an attributes sampling plan. Thus, a variables acceptance-sampling plan that has the same protection as an attributes acceptance-sampling plan would require less sampling.</p> <p>A second advantage is that measurement data usually provide more information about the manufacturing process or the lot than do attributes data. Generally, numerical measurements of quality characteristics are more useful than simple classification of the item as defective or nondefective. A final point to be emphasized is that when acceptable quality levels are very small, the sample sizes required by attributes sampling plans are very large. Under these circumstances, there may be significant advantages in switching to variables measurement. Thus, as many manufacturers begin to emphasize allowable numbers of defective parts per million, variables sampling becomes very attractive.</p> <p>Variables sampling plans have several disadvantages. Perhaps the primary disadvantage is that the distribution of the quality characteristic must be known. Furthermore, most standard variables acceptance-sampling plans assume that the distribution of the quality characteristic is normal. If the distribution of the quality characteristic is not normal, and a plan based on the normal assumption is employed, serious departures from the advertised risks of accepting or rejecting lots of given quality may be experienced.</p> <p>The second disadvantage of variables sampling is that a separate sampling plan must be employed for each quality characteristic that is being inspected. For example, if an item is inspected for four quality characteristics, it is necessary to have four separate variables inspection sampling plans; under attributes sampling, one attributes sampling plan could be employed. Finally, it is possible that the use of a variables sampling plan will lead to rejection of a lot even though the actual sample inspected does not contain any defective items. Although this does not happen very often, when it does occur it usually causes considerable unhappiness in both the suppliers' and the consumers' organizations, particularly if rejection of the lot has caused a manufacturing facility to shut down or operate on a reduced production schedule.</p>	
16.	<p>Explain the randomized block design and its applications in industrial experiments.</p> <p><b>Let there be <math>k</math> treatments. Each of the treatments is replicated the same number of times in this design. Let <math>r</math> denote the number of replications of each treatment. The total number of experimental units is, therefore, <math>kr</math>. These units are arranged into <math>r</math> groups, each of size <math>k</math>. The error control measure in this design consists of making the units in each of these groups homogeneous. These groups are commonly known as blocks and the experimental units in the blocks are known as plots.</b></p> <p><b>The data collected from experiments with randomized block designs form a two-way classification, that is, classified according to the levels of two factors, viz., blocks and treatments. There are <math>kr</math> cells in the two-way table with one observation in each cell. The data are orthogonal and therefore the design is called an <i>orthogonal design</i>.</b></p> <p><b>We take the model</b></p> $Y_{ij} = \mu + t_i + b_j + e_{ij} \quad \begin{matrix} (i = 1, 2, \dots, k) \\ (j = 1, 2, \dots, r) \end{matrix}$ <p><b>where <math>Y_{ij}</math> denotes the random variable corresponding to the observation <math>y_{ij}</math> from <math>i</math>th treatment in <math>j</math>th block, <math>\mu</math>, <math>t_i</math>, <math>b_j</math> are respectively the general mean, effect of the <math>i</math>th treatment and effect of the <math>j</math>th block as explained</b></p>	5 Marks

Let

$$\sum_j y_{ij} = T_i (i = 1, 2, \dots, k)$$

= observation total of *i*th treatment

and

$$\sum_j y_{ij} = B_j (j = 1, 2, \dots, r)$$

= observation total of *j*th block.

These are the marginal totals of the two-way data table.

Let further

$$\sum_i T_i = \sum_j B_j = G.$$

We shall call  $G^2/rk$  as *correction factor* denoted by C.F.

$$\text{Sum of squares due to treatment} = \sum_i \frac{T_i^2}{r} - \text{C.F.}$$

$$\text{Sum of squares due to blocks} = \sum_j \frac{B_j^2}{k} - \text{C.F.}$$

$$\text{Total sum of squares} = \sum_{ij} y_{ij}^2 - \text{C.F.}$$

#### Analysis of Variance of a Randomized Block Design

Sources of variation	Degrees of freedom (d.f.)	Sum of squares (s.s.)	Mean squares (m.s.) = $\frac{\text{s.s.}}{\text{d.f.}}$	<i>F</i>
Blocks	$r - 1$	$\sum_j \frac{B_j^2}{k} - \text{C.F.}$		
Treatments	$k - 1$	$\sum_i \frac{T_i^2}{r} - \text{C.F.}$	$s_i^2$	$\frac{s_i^2}{s^2}$
Error	$(r - 1)(k - 1)$	by subtraction	$s^2$	
<b>Total</b>	$rk - 1$	$\sum_{ij} y_{ij}^2 - \text{C.F.}$		

17. Identify the survivor function, failure rate function and mean time to failure (MTTF) for Weibull distribution.

5 Marks

The Weibull distribution is one of the most widely used life distributions in reliability analysis. The distribution is named after the Swedish professor Waloddi Weibull (1887–1979) who developed the distribution for modeling the strength of materials. The Weibull distribution is very flexible, and can, through an appropriate choice of parameters, model many types of failure rate behaviors.

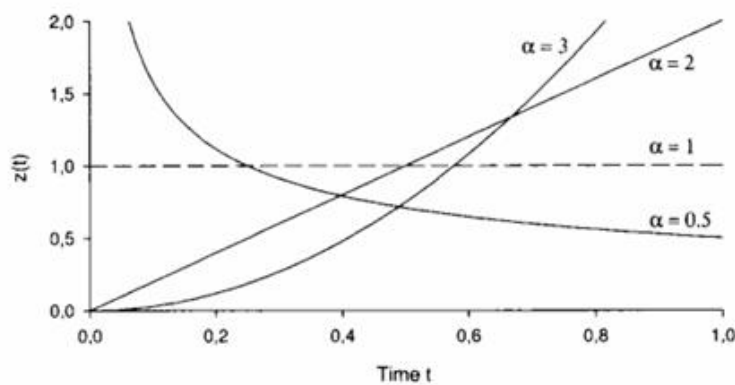
The time to failure  $T$  of an item is said to be Weibull distributed with parameters  $\alpha (> 0)$  and  $\lambda (> 0)$  [ $T \sim \text{Weibull}(\alpha, \lambda)$ ] if the distribution function is given by

$$F(t) = \Pr(T \leq t) = \begin{cases} 1 - e^{-(\lambda t)^\alpha} & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.50)$$

The corresponding probability density is

$$f(t) = \frac{d}{dt}F(t) = \begin{cases} \alpha\lambda^\alpha t^{\alpha-1} e^{-(\lambda t)^\alpha} & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.51)$$

where  $\lambda$  is a *scale* parameter, and  $\alpha$  is referred to as the *shape* parameter. Note that when  $\alpha = 1$ , the Weibull distribution is equal to the exponential distribution. The probability density function  $f(t)$  is illustrated in Fig. 2.15 for selected values of  $\alpha$ .



**Fig. 2.16** Failure rate function of the Weibull distribution,  $\lambda = 1$ .

The survivor function is

$$R(t) = \Pr(T > 0) = e^{-(\lambda t)^\alpha} \quad \text{for } t > 0 \quad (2.52)$$

and the failure rate function is

$$z(t) = \frac{f(t)}{R(t)} = \alpha\lambda^\alpha t^{\alpha-1} \quad \text{for } t > 0 \quad (2.53)$$

When  $\alpha = 1$ , the failure rate is constant; when  $\alpha > 1$ , the failure rate function is increasing; and when  $0 < \alpha < 1$ ,  $z(t)$  is decreasing. When  $\alpha = 2$ , the resulting

From (2.52) it follows that

$$R\left(\frac{1}{\lambda}\right) = \frac{1}{e} \approx 0.3679 \quad \text{for all } \alpha > 0$$

Hence  $\Pr(T > 1/\lambda) = 1/e$ , independent of  $\alpha$ . The quantity  $1/\lambda$  is sometimes called the *characteristic* lifetime. The mean time to failure is

$$\text{MTTF} = \int_0^\infty R(t) dt = \frac{1}{\lambda} \Gamma\left(\frac{1}{\alpha} + 1\right) \quad (2.54)$$

18. The following table presents the number of nonconformities observed in 26 successive samples of 100 printed circuit boards. Note that, for reasons of convenience, the inspection unit is defined as 100 boards. Set up a c chart for these data. 5 Marks

Sample Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of Nonconformities	21	24	16	12	15	5	28	20	31	25	20	24	16
Sample Number	14	15	16	17	18	19	20	21	22	23	24	25	26
Number of Nonconformities	19	10	17	13	22	18	39	30	24	16	18	17	15

Since the 26 samples contain 516 total nonconformities, we estimate  $c$  by

$$\bar{c} = \frac{516}{26} = 19.85$$

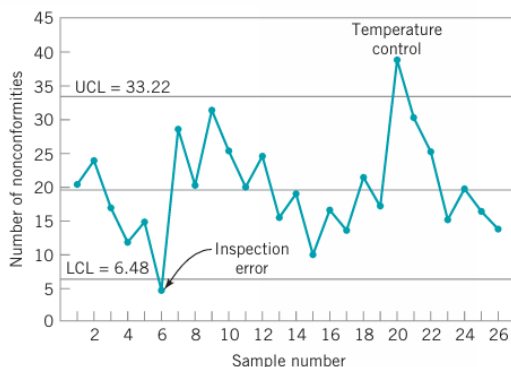
Therefore, the trial control limits are given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.85 + 3\sqrt{19.85} = 33.22$$

$$\text{Center line} = \bar{c} = 19.85$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.85 - 3\sqrt{19.85} = 6.48$$

The control chart is shown in Fig. 7.12. The number of observed nonconformities from the preliminary samples is plotted on this chart. Two points plot outside the control limits, samples 6 and 20. Investigation of sample 6 revealed that a new inspector had examined the boards in this sample and that he did not recognize several of the types of nonconformities that could have been present. Furthermore, the unusually large number of nonconformities in sample 20 resulted from a temperature control problem in the wave soldering machine, which was subsequently repaired. Therefore, it seems reasonable to exclude these two



■ FIGURE 7.12 Control chart for nonconformities for Example 7.3.

samples and revise the trial control limits. The estimate of  $c$  is now computed as

$$\bar{c} = \frac{472}{24} = 19.67$$

and the revised control limits are

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 19.67 + 3\sqrt{19.67} = 32.97$$

$$\text{Center line} = \bar{c} = 19.67$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 19.67 - 3\sqrt{19.67} = 6.36$$

These become the standard values against which production in the next period can be compared.

Twenty new samples, each consisting of one inspection unit (i.e., 100 boards), are subsequently collected. The number of nonconformities in each sample is noted and recorded in Table 7.8. These points are plotted on the control chart in Fig. 7.13. No lack of control is indicated; however, the number of nonconformities per board is still unacceptably high. Further action is necessary to improve the process.

19. Analyze the 2<sup>m</sup> factorial design and explain its importance in experiments. 5 Marks

Analysis Procedure for Factorial Designs  
 1. Estimate the factor effects  
 2. Form preliminary model  
 3. Test for significance of factor effects  
 4. Analyze residuals  
 5. Refine model, if necessary  
 6. Interpret result

Explain with example

**SECTION - C (4X10 = 40)**

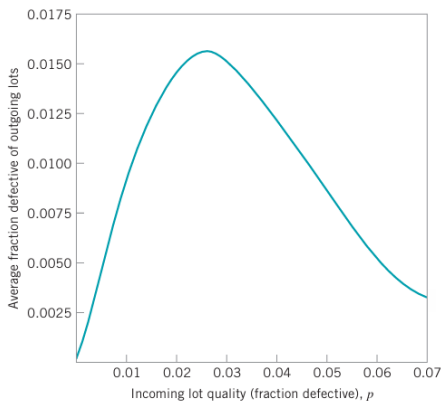
**Answer ANY FOUR Questions**

<p>20.</p>	<p>Explain the control charts of <math>\bar{X}</math> and R in Statistical quality control with examples.</p> <p>Suppose that a quality characteristic is normally distributed with mean <math>\mu</math> and standard deviation <math>\sigma</math>, where both <math>\mu</math> and <math>\sigma</math> are known. If <math>x_1, x_2, \dots, x_n</math> is a sample of size <math>n</math>, then the average of this sample is</p> $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ <p>and we know that <math>\bar{x}</math> is normally distributed with mean <math>\mu</math> and standard deviation <math>\sigma_{\bar{x}} = \sigma/\sqrt{n}</math>. Furthermore, the probability is <math>1 - \alpha</math> that any sample mean will fall between</p> $\mu + Z_{\alpha/2}\sigma_{\bar{x}} = \mu + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad \text{and} \quad \mu - Z_{\alpha/2}\sigma_{\bar{x}} = \mu - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \quad (6.1)$ $R = x_{\max} - x_{\min}$ <p>Let <math>R_1, R_2, \dots, R_m</math> be the ranges of the <math>m</math> samples. The average range is</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <math display="block">\bar{R} = \frac{R_1 + R_2 + \dots + R_m}{m} \quad (6.3)</math> </div> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="background-color: black; color: white; padding: 5px;"><b>Control Limits for the <math>\bar{x}</math> Chart</b></td> <td style="background-color: black; color: white; padding: 5px;"><b>Control Limits for the R Chart</b></td> </tr> <tr> <td style="text-align: center; padding: 5px;"><math>UCL = \bar{x} + A_2\bar{R}</math></td> <td style="text-align: center; padding: 5px;"><math>UCL = D_4\bar{R}</math></td> </tr> <tr> <td style="text-align: center; padding: 5px;">Center line = <math>\bar{x}</math></td> <td style="text-align: center; padding: 5px;">Center line = <math>\bar{R}</math></td> </tr> <tr> <td style="text-align: center; padding: 5px;"><math>LCL = \bar{x} - A_2\bar{R}</math></td> <td style="text-align: center; padding: 5px;"><math>LCL = D_3\bar{R}</math></td> </tr> </table> <p>Explain the operations of control charts of <math>\bar{X}</math> and R with example</p>	<b>Control Limits for the <math>\bar{x}</math> Chart</b>	<b>Control Limits for the R Chart</b>	$UCL = \bar{x} + A_2\bar{R}$	$UCL = D_4\bar{R}$	Center line = $\bar{x}$	Center line = $\bar{R}$	$LCL = \bar{x} - A_2\bar{R}$	$LCL = D_3\bar{R}$	<p>10 Marks</p>
<b>Control Limits for the <math>\bar{x}</math> Chart</b>	<b>Control Limits for the R Chart</b>									
$UCL = \bar{x} + A_2\bar{R}$	$UCL = D_4\bar{R}$									
Center line = $\bar{x}$	Center line = $\bar{R}$									
$LCL = \bar{x} - A_2\bar{R}$	$LCL = D_3\bar{R}$									
<p>21.</p>	<p>Construct OC curve, AOQ curve and ATI curve for the single sampling plan <math>N=10,000</math>, <math>n=89</math>, and <math>c=2</math>, and that the incoming lots are of quality <math>p=0.01</math>.</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> <math display="block">AOQ = \frac{P_a p(N - n)}{N} \quad (15.4)</math> </div> <p>To illustrate the use of equation (15.4), suppose that <math>N = 10,000</math>, <math>n = 89</math>, and <math>c = 2</math>, and that the incoming lots are of quality <math>p = 0.01</math>. Now at <math>p = 0.01</math>, we have <math>P_a = 0.9397</math>, and the AOQ is</p> $  \begin{aligned}  AOQ &= \frac{P_a p(N - n)}{N} \\  &= \frac{(0.9397)(0.01)(10,000 - 89)}{10,000} \\  &= 0.0093  \end{aligned}  $	<p>10 Marks</p>								

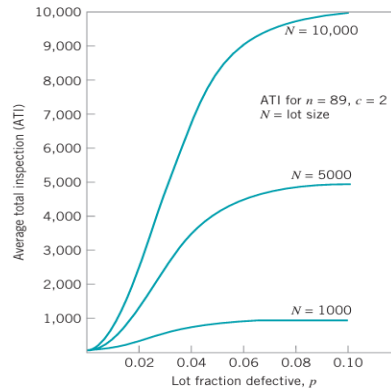
$$ATI = n + (1 - P_a)(N - n) \quad (15.6)$$

To illustrate the use of equation (15.6), consider our previous example with  $N = 10,000$ ,  $n = 89$ ,  $c = 2$ , and  $p = 0.01$ . Then, since  $P_a = 0.9397$ , we have

$$\begin{aligned} ATI &= n + (1 - P_a)(N - n) \\ &= 89 + (1 - 0.9397)(10,000 - 89) \\ &= 687 \end{aligned}$$



■ FIGURE 15.11 Average outgoing quality curve for  $n = 89$ ,  $c = 2$ .



■ FIGURE 15.12 Average total inspection (ATI) curves for sampling plan  $n = 89$ ,  $c = 2$ , for lot sizes of 1000, 5000, and 10,000.

22. Explain the concept of two-sided specifications and their significance in variable sampling plans.

CO3

23. Four different methods of preparing concrete mixtures A, B, C, D were tested, these methods consist of two different mixture ratios of cement to water and two blending durations. The four methods (treatments) were blocked in four batches and four days, according to a Latin square design. The concrete was poured to cubes and tested for compressive strength [kg/cm<sup>2</sup>] after 7 days of storage in special rooms with 20°C temperature and 50% relative humidity.

CO4

The results are:

Batches Days	1	2	3	4
1	A 312	B 299	C 315	D 290
2	C 295	A 317	D 313	B 300
3	B 295	D 298	A 312	C 315
4	D 313	C 314	B 299	A 300

Test whether the differences between the strength values of different treatments are significant?

Correction Factor (CF):  $\frac{T^2}{N} = \frac{4887^2}{16} = 1492659.0625$

Total Sum of Squares (SSTotal):  $\sum X^2 - CF$

$SSTotal = (312^2 + 299^2 + \dots + 300^2) - 1492659.0625 = 1496055 - 1492659.0625 = 3395.9375$

	$SS_{Batches} = \frac{1216^2 + 1225^2 + 1220^2 + 1226^2}{4} - 1492659.0625 = 1492724.25 - 1492659.0625 = 65.1875$ $SS_{Days} = \frac{1215^2 + 1228^2 + 1239^2 + 1205^2}{4} - 1492659.0625 = 1493027.75 - 1492659.0625 = 368.6875$ $SS_{Treatments} = \frac{1241^2 + 1193^2 + 1239^2 + 1214^2}{4} - 1492659.0625 = 1495034.25 - 1492659.0625 = 2375.1875$ $SSE = 3395.9375 - 65.1875 - 368.6875 - 2375.1875 = 586.875$ <ul style="list-style-type: none"> <li>◦ <math>MS_{Batches} = \frac{SS_{Batches}}{df_{Batches}} = \frac{65.1875}{3} = 21.729</math></li> <li>◦ <math>MS_{Days} = \frac{SS_{Days}}{df_{Days}} = \frac{368.6875}{3} = 122.896</math></li> <li>◦ <math>MS_{Treatments} = \frac{SS_{Treatments}}{df_{Treatments}} = \frac{2375.1875}{3} = 791.729</math></li> <li>◦ <math>MSE = \frac{SSE}{df_{Error}} = \frac{586.875}{6} = 97.8125</math></li> </ul> <ul style="list-style-type: none"> <li>• <b>F-statistic for Treatments:</b> <math>F_{Treatments} = \frac{MS_{Treatments}}{MSE} = \frac{791.729}{97.8125} = 8.094</math></li> </ul> <p>For a significance level of <math>\alpha = 0.05</math> and degrees of freedom <math>df_1 = 3</math> (for treatments) and <math>df_2 = 6</math> (for error), the critical F-value from an F-distribution table is approximately <math>F_{critical} = 4.76</math>.</p>	
24.	<p>Determine the survivor function, failure rate function, mean time to failure (MTTF) and mean residual life (MRL) for Exponential distribution.</p> <p><b>Exponential Distribution:</b></p> <p>Consider an item that is put into operation at time <math>t = 0</math>. The time to failure <math>T</math> of the item has probability density function</p> $f(t) = \begin{cases} \lambda e^{-\lambda t} & \text{for } t > 0, \lambda > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.27)$ <p>This distribution is called the <i>exponential distribution</i> with parameter <math>\lambda</math>, and we sometimes write <math>T \sim \exp(\lambda)</math>.</p> <p>The reliability (survivor) function of the item is</p> $R(t) = \Pr(T > t) = \int_t^{\infty} f(u) du = e^{-\lambda t} \quad \text{for } t > 0 \quad (2.28)$ <p>The probability density function <math>f(t)</math> and the survivor function <math>R(t)</math> for the exponential distribution are illustrated in Fig. 2.8. The mean time to failure is</p> $MTTF = \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda} \quad (2.29)$ <p>and the variance of <math>T</math> is</p> $\text{var}(T) = \frac{1}{\lambda^2}$ <p>The probability that an item will survive its mean time to failure is</p> $R(MTTF) = R\left(\frac{1}{\lambda}\right) = e^{-1} \approx 0.3679$ <p>The failure rate function is</p> $z(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \quad (2.30)$	10 Marks

Accordingly, the failure rate function of an item with exponential life distribution is constant (i.e., independent of time). By comparing with Fig. 2.5, we see that this indicates that the exponential distribution may be a realistic life distribution for an item during its useful life period, at least for certain types of items.

The results (2.29) and (2.30) compare well with the use of the concepts in everyday language. If an item on the average has  $\lambda = 4$  failures/year, the MTTF of the item is 1/4 year.

Consider the conditional survivor function (2.17)

$$\begin{aligned} R(x | t) &= \Pr(T > t + x | T > t) = \frac{\Pr(T > t + x)}{\Pr(T > t)} \\ &= \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x} = \Pr(T > x) = R(x) \end{aligned} \quad (2.31)$$

The survivor function of an item that has been functioning for  $t$  time units is therefore equal to the survivor function of a new item. A new item, and a used item (that is still functioning), will therefore have the same probability of surviving a time interval of length  $t$ . The MRL for the exponential distribution is

$$\text{MRL}(t) = \int_0^\infty R(x | t) dx = \int_0^\infty R(x) dx = \text{MTTF}$$

The  $\text{MRL}(t)$  of an item with exponential life distribution is hence equal to its MTTF irrespective of the age  $t$  of the item. The item is therefore *as good as new* as long as it is functioning, and we often say that the exponential distribution has *no memory*.

25.	<p>Explain the role of total quality management in modern industry</p> <p>Total Quality Management. Total quality management (TQM) is a strategy for implementing and managing quality improvement activities on an organizationwide basis. TQM began in the early 1980s, with the philosophies of Deming and Juran as the focal point. It evolved into a broader spectrum of concepts and ideas, involving participative organizations and work culture, customer focus, supplier quality improvement, integration of the quality system with business goals, and many other activities to focus all elements of the organization around the quality improvement goal. Typically, organizations that have implemented a TQM approach to quality improvement have quality councils or high-level teams that deal with strategic quality initiatives, workforce-level teams that focus on routine production or business activities, and cross-functional teams that address specific quality improvement issues. TQM has only had moderate success for a variety of reasons, but frequently because there is insufficient effort devoted to widespread utilization of the technical tools of variability reduction. Many organizations saw the mission of TQM as one of training. Consequently, many TQM efforts engaged in widespread training of the workforce in the philosophy of quality improvement and a few basic methods. This training was usually placed in the hands of human resources departments, and much of it was ineffective. The trainers often had no real idea about what methods should be taught, and success was usually measured by the percentage of the workforce that had been "trained," not by whether any measurable impact on business results had been achieved. Some general reasons for the lack of conspicuous success of TQM include (1) lack of topdown, high-level management commitment and involvement; (2) inadequate use of statistical methods and insufficient recognition of variability reduction as a prime objective; (3) general as opposed to specific business-results-oriented objectives; and (4) too much emphasis on widespread training as opposed to focused technical education.</p>	10 Marks
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