Scheme of Valuation

Fuzzy Sets and their Applications.

24PMSMTIE2

Section A — 1 Mark

1. Equality on fuzzy subsets:

$$\mu_A(x) = \mu_B(x).$$

2. Generalized relative Hamming Distance:

$$\delta(\tilde{A}, \tilde{B}) = \frac{d(\tilde{A}, \tilde{B})}{n} = \frac{1}{n}\sum_{i=1}^{n} |\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)|.$$

3. Index of fuzziness for a product.

$$\gamma(\tilde{A}) = \frac{4}{N}\sum_{i=1}^{N} \mu_{\tilde{A}\cdot\bar{\tilde{A}}}(x_i)$$

4. Normal $h(\tilde{R}) = 1$ SubNormal $h(\tilde{R}) < 1$.

5. Algebric sum of two relations.

$$\mu_{\tilde{R}\hat{+}\tilde{Q}}(x,y) = \mu_{\tilde{R}}(x,y) + \mu_{\tilde{Q}}(x,y) - \mu_{\tilde{R}}(x,y)\cdot\mu_{\tilde{Q}}(x,y).$$

6. Fuzzy equivalence Relation:

Transitive, reflexive, symmetric.

7. Perfect Antisymmetry:

$\forall\, x, y \in X \times X$ with $x \neq y$:

$$\mu_{\tilde{R}}(x,y) > 0 \Rightarrow \mu_{\tilde{R}}(y,x) = 0$$

8. min-max distance in a resemblance relation.

$$d_{\hat{\underset{\sim}{R}}}(x,y) = 1-\mu_{\underset{\sim}{R}}(x,y)$$

9. Mixed functions of fuzzy variables.

$a, b, \ldots \in [0,1]$ may be submitted to operations other than $\wedge$, $\vee$, and $-$ in order to form.

10. Reduced polynomial form.

Any polynomial form with respect to $\vee$ that does not contain a maximal monomial in $\wedge$ will be said to be a RPF.

11. fuzzy monoid.

Any fuzzy groupoid that is associative and has an identity will be called a fuzzy monoid

12. groupoid.

An ordered pair formed by a set E and an internal law of composition $*$ defined on this set is called a groupoid $(E, *)$

Section - B 5 Marks.

13. Decomposition thm on fuzzy subset.

Any fuzzy subset $\underset{\sim}{A}$ may be decomposed in the following form, clearly as products of ordinary subsets by the coefficients α_i — 1 Mark

3

$$\underset{\sim}{A} = \underset{\alpha_i}{MAX}\left[\alpha_1 \cdot A_{\alpha_1}, \alpha_2 \cdot A_{\alpha_2} \cdots, \alpha_n, A_{\alpha_n}\right],$$

$0 < \alpha_i \leq 1, \ i = 1, 2 \cdots n.$

<u>proof!</u> — 2 Marks

$\mu_{A_{\alpha_i}}(x) = 1$ if $\mu_A(x) \geq \alpha_i$

$= 0$ if $\mu_A(x) < \alpha_i$

$\mu(x)$

$\underset{\alpha_i}{Max}[\alpha_i \cdot A_{\alpha_i}] = \underset{\alpha_i}{MAX}[\alpha_i, A_{\alpha_i}]$

$= \underset{\alpha_i \leq \mu_A(x)}{MAX}[\alpha_i]$

with example! $= \mu_A(x)$. 2 Marks.

14. prove that $\mu_R^k(x,y) = \ell_R^*(x,y)$. 5 Marks.

15 <u>T.P!</u> $R^2 = R$.

$\forall x \ \mu_R(x,x) = 1$

$\because \underset{\sim}{R}^2 = \underset{\sim}{R} \circ \underset{\sim}{R}$

$\mu_{R^2}(x,z) = \underset{y}{\vee}[\mu_{\underset{\sim}{R}}(x,y) \wedge \mu_{\underset{\sim}{R}}(y,z)]$

$\mu_{\underset{\sim}{R}}(x,x) = \mu_R(z,z) = 1$ Reflexivity.

$\therefore R$ is transitive and $\mu_{\underset{\sim}{R}}(x,z) \geq \underset{y}{\vee}[\mu_R(x,y) \wedge \mu_{\underset{\sim}{R}}(y,z)]$

— 3 marks

We want to p.T $R^K = R \;\forall\; K \geq 1$

This requires $R^K \subseteq R$ & $R \subseteq R^K \Rightarrow R^K = R$.

↳ 2 marks.

16. Decomposition thm for a Similitude Relation:

Statement:

Let $\mathbb{R}$ be a similitude relation in $E \times E$.

Then $\mathbb{R}$ may be decomposed in the form.

$$\underset{\sim}{R} = \bigvee_{\alpha} \alpha \cdot R_\alpha \quad 0 < \alpha \leq 1$$

with $\alpha_1 > \alpha_2 \Rightarrow R_2 \supset R_1$ — 2 marks.

Where the R_α are equivalence relations in the sense of ordinary set theory and $\alpha \cdot R_\alpha$ indicates that all the elements of the ordinary relation R_α are multiplied by α.

proof: 3 marks.

17. Commutative $a \wedge b = b \wedge a$; $a \vee b = b \vee a$.

Associativity: $(a \wedge b) \wedge c = a \wedge (b \wedge c)$

$(a \vee b) \vee c = a \vee (b \vee c)$

idempotence : $a \wedge a = a$

$a \vee a = a$

5 marks.

distributivity : $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$(\overline{\underset{\sim}{\bar{a}}}) = a$

and De Morgan's $\overline{a \wedge b} = \bar{a} \vee \bar{b}$

$\overline{a \vee b} = \bar{a} \wedge \bar{b}$

18. Law of external Composition.

$x \in E_1,\ y \in E_2,\ z \in E_3$ — 2 marks.

$E_1 \times E_2 \rightarrow E_3$ is called Law of External Composition.

Examples: — 3 marks

$E_1 = E_2 = R$

$P(E)$ powerset, intersection, union etc.

$E_1 = E_2 = R^+$

$E_1 = E_2 =$ free vectors in a plane.

19. Commutativity: $A * B = B * A$.

$$\mu_{A*B}(x) = \mu_A(x) \odot \mu_B(x)$$

— 2½ marks.

Associativity:

$$(A * B) * C = A * (B * C)$$ — 2½ marks.

$$(\mu_A(x) \odot \mu_B(x)) \odot \mu_C(x) = (\mu_A(x) \odot (\mu_B(x) \odot \mu_C(x)))$$

Section - C 10 Marks.

20 Proof:

$$\sum_{\substack{i,j=1 \\ i \neq j}}^{k} (m_i n_j - m_j n_i)^2 \geq 0.$$

— 1 mark

Developing this sum squares, we have

$$\sum_{\substack{i,j=1\\ i\neq j}}^{k} m_i^2 n_j^2 + \sum_{i=1}^{k} m_i^2 n_i^2 \geq \sum_{i=1}^{k} m_i^2 n_i^2 + \sum_{\substack{i,j=1\\ i\neq j}}^{k} 2 m_i n_i m_j n_j$$ – 5 marks.

Finally $\sqrt{\sum_{i=1}^{k} m_i^2} + \sqrt{\sum_{i=1}^{k} n_i^2} \geq \sqrt{\sum_{i=1}^{k} (m_i + n_i)^2}$

$\because$ $m_i + n_i \geq p_i$ will reach the Result.

↳ 4 marks

21 i) Transitive closure of any fuzzy binary relation is a transitive binary relation.

<u>proof:</u>

$$\hat{R}^2 = \hat{R} \cdot \hat{R} = R^2 \cup R^3 \cup R^4 \cup \cdots$$

$(R \supset R^2) \Leftrightarrow (R = \hat{R}) \Leftrightarrow (R$ is transitive$)$

$(R = R^2) \Leftrightarrow (R = \hat{R}) \Leftrightarrow (R$ is " $)$

↳ 5 marks.

ii) <u>proof:</u>

$$\hat{R} = R \cup R^2 \cup \cdots \cup R^k \cup R^{k+1} \cup R^{k+2} \cup \cdots$$

$$= R \cup R^2 \cup \cdots \cup R^k \cup R^k \cup R^k \cup \cdots$$

$$= R \cup R^2 \cup \cdots \cup R^k.$$

↳ 5 marks.

22. <u>proof:</u>

<u>T.P:</u> $c \geq a = b$ (or) $a \geq b = c$ or $b \geq c = a$

$\therefore$ $c \geq a \wedge b$, $b \geq c \wedge a$, $a \geq b \wedge c$

The inequalities give

If $a = b$: $c \geq a \wedge b$, $b = c \wedge a$, $a = b \wedge c$

If $b = c$: $c = a \wedge b$, $b = c \wedge a$, $a \geq b \wedge c$

If $c = a$: $c = a \wedge b$, $b \geq c \wedge a$, $a = b \wedge c$.

— 10 marks.

23. Proof:

$$\hat{\underset{\sim}{R}} = \underset{\sim}{R} \cup \underset{\sim}{R}^2 \cup \underset{\sim}{R}^3 \cup \ldots$$

Min-max transitive will expressed by

$$\check{\underset{\sim}{R}} = \underset{\sim}{R} \cap (\underset{\sim}{R} * \underset{\sim}{R}) \cap (\underset{\sim}{R} * \underset{\sim}{R} * \underset{\sim}{R}) \cap \ldots$$

Let $\underset{\sim}{R}$ be a resemblance Relation, $\hat{\underset{\sim}{R}}$ is a similitude relation, $\bar{\underset{\sim}{R}}$ is a dissemblance relation and $\check{\underset{\sim}{R}}$ is a dissimilitude relation. We show that $\bar{\hat{\underset{\sim}{R}}} = \check{\underset{\sim}{R}}$

— 10 marks.

24 Sheffer operator

$$a \mid b = \overline{a \cdot b}$$

$$= \bar{a} + \bar{b}$$

5 marks

$$a + b = \bar{a} \mid \bar{b} = (a \mid a)(b \mid b),$$

$$a \cdot b = \overline{a \mid b} = (a \mid b) \mid (a \mid b)$$

$$\bar{a} = a \mid a$$

Peirce Operator:

$$a \downarrow b = \overline{a+b}$$
$$= \bar{a} \cdot \bar{b}$$

5 marks.

$$a+b = \overline{a \downarrow b} = (a \downarrow b) \downarrow (a \downarrow b)$$
$$a \cdot b = \bar{a} \downarrow \bar{b} = (a \downarrow a) \downarrow (b \downarrow b)$$
$$\bar{a} = a \downarrow a.$$

and discuss their significance in Boolean Algebra.

25. Exponential fuzzy integers.

Consider $E = R^+$

$$\underset{\sim}{I}_1 \Rightarrow \mu_{\underset{\sim}{I}_1}(x) = \lambda e^{-\lambda x}, \; x \in R^+$$

10 marks.

$$\underset{\sim}{I}_2 \Rightarrow \mu_{\underset{\sim}{I}_2}(x) = \lambda^2 x e^{-\lambda x}$$

$$\underset{\sim}{I}_3 \Rightarrow \mu_{\underset{\sim}{I}_3}(x) = \frac{\lambda^3 x^2 e^{-\lambda x}}{2}$$

$$\underset{\sim}{I}_n \Rightarrow \mu_{\underset{\sim}{I}_n}(x) = \lambda^n x^{n-1} e^{-\lambda x} / n!$$

$$\underset{x}{MAX}\, \mu_{I_n}(x) = \lambda \frac{(n-1)^{n-1} e^{-(n-1)}}{(n-1)!}$$

The monoid $I_0, I_1, I_2 \ldots I_n \ldots$ is isomorphic to that of the natural numbers.

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