Answer Key

24PMSMTIE2 - Fuzzy Sets and their Applications

Section-A ($10 \times 1 = 10$ marks) (Answer any TEN)

1. Let E be a set & M its associated membership set and let $\underset{\sim}{A}$ and $\underset{\sim}{B}$ be two fuzzy subsets of E; $\underset{\sim}{A}$ is included in $\underset{\sim}{B}$ if $\forall x \in E$: $\mu_{\underset{\sim}{A}}(x) < \mu_{\underset{\sim}{B}}(x)$ denoted by $\underset{\sim}{A} \subset \underset{\sim}{B}$

2. Difference: $\underset{\sim}{A} - \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$

3. Idempotence: $\underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$ & $\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}$.

4. Berge Graph: $E_1 = E_2 = E$ countable and is formed by the subset of ordered pairs $(x, y) \in G \subset E \times E$; $G \cap \overline{G} = \phi$; $G \cup \overline{G} = E \times E$

5. Let $\underset{\sim}{R}$ and $\underset{\sim}{L}$ be two fuzzy relations such that $\forall (x, y) \in E_1 \times E_2$; $\mu_{\underset{\sim}{R}}(x, y) \leq \mu_{\underset{\sim}{L}}(x, y)$; say that $\underset{\sim}{L}$ is an envelope of $\underset{\sim}{R}$

6. Transitive relation that is not reflexive is called semi pre order relation

7. Perfect Anti symmetric: $\forall (x, y) \in E \times E$ with $x \neq y$; $\mu_{\underset{\sim}{R}}(x, y) > 0 \Rightarrow \mu_{\underset{\sim}{R}}(y, x) = 0$

8. Dissimilitude: (i) $\forall (x, y)\,(y, z)\,(z, x) \in E \times E$; $\mu_{\underset{\sim}{R}}(x, z) \geq \vee [\mu_{\underset{\sim}{R}}(x, y) \wedge \mu_{\underset{\sim}{R}}(y, z)]$ transitivity
   (ii) $\forall (x, x) \in E \times E$; $\mu_{\underset{\sim}{R}}(x, x) = 1$ (Reflexivity)
   (iii) $\forall (x, y) \in E \times E$; $\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{R}}(y, x)$ (symmetry)

9. $f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\bar{a} \vee \bar{b}) \wedge (\bar{a} \vee c) \wedge (b \vee \bar{b}) \wedge (b \vee c) \wedge (c \vee \bar{c}) \wedge (\bar{b} \vee \bar{c})$

10. f_1 and f_2 are equal if they produce the same table of values through enumeration of all possible values

11. Identity element: $\forall a \in E$: $e * a = a$; Left Identity
    $\forall a' \in E$; $a * e' = a'$ Right Identity.

12. Fuzzy Submonoid: Let $(P(E), *)$ be a fuzzy monoid and $\Delta \subset P(E)$ be closed for the Law $*$ then Δ will be called a fuzzy submonoid of $(P(E), *)$

Section-B ($5 \times 5 = 25$ marks) (Answer any FIVE)

13. (a) $A \oplus B = (A \cap \bar{B}) \cup (\bar{A} \cap B) = \{ x_1/0.3, x_2/0.7, x_3/0.5, x_4/0.2, x_5/1, x_6/0.5, x_7/0.4 \}$ — 1.25 marks

    (b) $\delta(A, B) = 0.38571$ — 1.25 marks

    (iii) $e(A, B) = 1.3228$ — 1.25 marks

    (iv) $E(A, B) = 0.4999$ — 1.25 marks

24PMSMTIE2 - Fuzzy sets and Their Applications

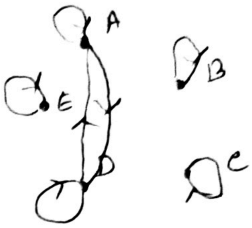
14.

$$\vec{R}: \quad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & e^{-k} & e^{-4k} & e^{-9k} \;\cdots \\ 1 & e^{-k} & 1 & e^{-k} & e^{-4k} \;\cdots \\ 2 & e^{-4k} & e^{-k} & 1 & e^{-k} \;\cdots \\ 3 & e^{-9k} & e^{-4k} & e^{-k} & 1 \;\cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \qquad \overrightarrow{R \circ R}: \quad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & e^{-k} & e^{-k} & e^{-4k} \;\cdots \\ 1 & e^{-k} & 1 & e^{-k} & e^{-k} \;\cdots \\ & e^{-k} & e^{-k} & 1 & e^{-k} \;\cdots \\ 2 & e^{-4k} & e^{-k} & e^{-k} & 1 \;\cdots \\ 3 & & & & \end{array}$$

$\underset{\sim}{R} \neq R^2$ (Not Transitive)

15. distance equal to 0

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 1 & 0 & 0 & 1 & 0 \\ B & 0 & 1 & 0 & 0 & 0 \\ C & 0 & 0 & 1 & 1 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 0 & \end{array}$$



draw a corresponding Transitive graph

Similarly distance $\leq 0.1,\ 0.2,\ 0.8$

16 fuzzy order Relation (i) Reflexive, (ii) transitive (iii) Antisymmetric

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 1 & 0.8 & 0 & 0 \\ B & 0.2 & 1 & 0 & 0 \\ C & 0.3 & 0.4 & 1 & 0.1 \\ D & 0 & 0 & 0 & 1 \end{array}$$

17. mixed operations; The variables $\underset{\sim}{a}, \underset{\sim}{b} \ldots \in [0,1]$ may be submitted to operations other than $\wedge$, $\vee$ and $\sim$ in order to form will be called mixed function of fuzzy variables Product $\underset{\sim}{a} \in [0,1]$; $\underset{\sim}{b} \in [0,1] \Rightarrow a.b \in [0,1]$

sum $\underset{\sim}{a} \hat{+} \underset{\sim}{b} = \underset{\sim}{a} + \underset{\sim}{b} - \underset{\sim}{a}.\underset{\sim}{b}$

$f(\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}) = (\underset{\sim}{a} \hat{+} \underset{\sim}{b}) \wedge (\underset{\sim}{b} \hat{+} \underset{\sim}{c}) \wedge (a \wedge c)$ is mixed function

18. Any function groupoid that is associative and Identity will be called fuzzy monoid

$(P(E), \hat{+})$ when $\mu_{A \hat{+} B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$

Associative with identity ϕ

$(P(E)\ \oplus)$ when $\mu_{A \oplus B}(x) = [\mu_A(x) \wedge 1 - \mu_B(x)] \vee [1 - \mu_A(x) . \wedge \mu_B(x)]$

Associative with identity ϕ

Fuzzy submonoid :- Let $[(P(E), *]$ be a fuzzy monoid and $\Delta \subset P(E)$ be closed for the law $*$ then Δ will be called a fuzzy submonoid of $(P(E), *)$

24 PMSMT1E2 - Fuzzy sets and their Applications.

19. Let E be reference set and $\underset{\sim}{A} \subset E$ fuzzy subset of E by P(E)
$\underset{\sim}{A} \in P(E)$ $n = card(E)$; $m = card(M)$ are finite ; P(E) is finite
Internal composition on $\underset{\sim}{P}(E)$ mapping from $\underset{\sim}{P}(E) \times \underset{\sim}{P}(E)$ into P(E)
$E\{A, B\}$; $m = \{0, \frac{1}{2}, 1\}$: $\underset{\sim}{P}(E) = \{(A|0)(B|0)\}\ \{(A|0)(B|\frac{1}{2})\},$
$\{(A|\frac{1}{2}),(B|0)\}\ \{(A|\frac{1}{2})(B|\frac{1}{2})\}, \{(A|0)(B|1)\}\ \{(A|1)(B|0)\}$
$\{(A|\frac{1}{2}),(B|1)\}\ \{(A|1),(B|\frac{1}{2})\}\ \{(A|1),(B|1)\}$
Write a table represent internal composition groupoid.

Section-C (4×10 = 40 marks) Answer any Four

20. Inequality $\sum_{i,j=1}^{k} (m_i n_j - m_j n_i)^2 \geq 0$
developing sum of square and Adding $\sum_{i=1}^{k} m_i^2 n_i^2$ we get

$$2\sqrt{\sum_{i=1}^{k} m_i^2} \cdot \sqrt{\sum_{i=1}^{k} n_i^2} \geq 2\sum_{i=1}^{k} m_i n_i$$

Adding $\sum m_i^2 + \sum n_i^2$ and simplifying

$$\sqrt{\sum_{i=1}^{k} m_i^2} + \sqrt{\sum_{i=1}^{k} n_i^2} \geq \sqrt{\sum_{i=1}^{k} p_i^2}$$

21. (i) Decomposition Theorem, Any fuzzy relation R may be decomposed in the form $\underset{\sim}{R} = V\alpha . R_\alpha$; $0 < \alpha \leq 1$

$$\mu R_\alpha(x, y) = 1 \ ; \ \text{if } \mu_{\underset{\sim}{R}}(x, y) \geq \alpha$$
$$= 0 \ ; \ \text{if } \mu R(x, y) < \alpha$$

R_α indicates ordinary relation R_α are multiplied by α.
Proof:- $\mu_{\underset{\alpha}{V} R_\alpha}(x, y) = \underset{\alpha}{V} \cdot \alpha \, \mu R_\alpha(x, y) = \mu_{\underset{\sim}{R}}(x, y)$

Explain with an example.

(ii) Let $\underset{\sim}{R}$ be any fuzzy binary relation ; If for some k $\underset{\sim}{R}^{k+1} = \underset{\sim}{R}^k$

$$\hat{\underset{\sim}{R}} = \underset{\sim}{R} \cup R^2 \cup \ldots \cup \underset{\sim}{R}^k$$

Proof:- $\hat{R} = R \cup R^2 \ldots \cup R^{k+1} \cup \ldots$
$= R \cup R^2 \ldots \cup R^k$

$R \subset E \times E$ where E is finite and $card(E) = n$ then

$$\hat{\underset{\sim}{R}} = \underset{\sim}{R} \cup R^2 \cup \ldots \cup R^n \ ; \ k \leq n.$$

2a PMSMT1E2 - Fuzzy Sets and their Applications

22. Theorem of decomposition for a Similitude relation; Let $\underset{\sim}{R}$ be a Similitude relation in $E \times E$, then $\underset{\sim}{R}$ may be decomposed in the form

$$\underset{\sim}{R} = \bigvee_{\alpha} \alpha \cdot R_\alpha \; ; \; 0 < \alpha \le 1 \quad \text{with } \alpha_1 > \alpha_2 \Rightarrow R_2 \supset R_1$$

Proof:- $\mu_{\underset{\sim}{R}}(x,x) = 1$, follows $(x,x) \in R_\alpha$; $\alpha \in [0,1]$ - Reflexivity

R_α has the property of symmetry, transitive

$\therefore R_\alpha$ is equivalence relation.

Conversely R_α is non-empty $(x,x) \in R_1$ and

$\mu_{\underset{\sim}{R}}(x,x) = 1 \; ; \; \forall x \in E$

then R is reflexive.

$\mu_{\underset{\sim}{R}}(x,y) = \bigvee_{\alpha} \alpha \cdot \mu_{R_\alpha}(x,y)$ it is evident that the

Symmetry of each $\underset{\sim}{R}$

Finally $\mu_{\underset{\sim}{R}}(x,y) = \alpha$; $\mu_{\underset{\sim}{R}}(y,z) = \beta$

$(x,y) \in R_{\alpha \wedge \beta}$; $(y,z) \in R_{\alpha \wedge \beta}$

$(x,z) \in R_{\alpha \wedge \beta}$.

$\mu_{\underset{\sim}{R}}(x,z) \geq \bigvee_y [\mu_{\underset{\sim}{R}}(x,y) \wedge \mu_{\underset{\sim}{R}}(y,z)]$ transitive.

23. Proof:- hypothesis $c \geq a \wedge b$ — ① ; $b \geq c \wedge a$ — ② ; $a \geq b \wedge c$ — ③

Suppose $c \geq b > a$ — ④ ; ① & ② are verified but ③ is not ; takes $b = a$ it verified.

$c \geq a > b$ — ⑤ ① & ③ are verified but ② is not ; take $a = b$ it is verified

$c \geq a = b$ ④ & ⑤ are contrary

Similarly $a \geq b = c$; $b \geq a = c$

It is necessary that atleast two of the values are equal

If $a = b$ $c \geq a \wedge b$; $b = c \wedge a$; $a = b \wedge c$

If $b = c$ $c = a \wedge b$; $b = c \wedge a$; $a \geq b \wedge c$

If $c = a$; $c = a \wedge b$; $b \geq c \wedge a$; $a = b \wedge c$

24. $f(a, b, c) = (a \wedge \bar{b}) \vee (\bar{a} \wedge c) \vee \bar{c}$

Interval $[0, 0.2[$

Hypothesis I : $a \wedge \bar{b} > \bar{a} \wedge c$; $a \wedge \bar{b} > \bar{c}$

$\Rightarrow 0 \leq a \wedge \bar{b} < 0.2$

$0 \leq \text{MIN}(a, 1-b) < 0.2$

$(a \geq 0 \text{ and } b \leq 1)$ and $(a < 0.2$ or/and $b > 0.8)$

Hypothesis II :- $\bar{a} \wedge c > a \wedge \bar{b}$; $\bar{a} \wedge c > \bar{c}$

$0 \leq \bar{a} \wedge c < 0.2$

$0 \leq \text{MIN}(1-a, c) < 0.2$

$(a \leq 1 \text{ and } c \geq 0)$ and $(a > 0.8$ or/and $c < 0.2)$

Hypothesis III :- $\bar{c} > a \wedge \bar{b}$; $\bar{c} > \bar{a} \wedge c$

$0 \leq \bar{c} < 0.2$

$0 \leq 1 - c < 0.2$

$0.8 < c < \phi.$

Similarly Interval $[0.2, 0.3[$, $[0.3, 1]$ will find out.

25. Exponential fuzzy integers :- $\mu_{I_1}(x) = \lambda e^{-\lambda x}$; $x \in R^+$

$\mu_{I_2}(x) = \mu_{I_1}(x) * \mu_{I_1}(x) = \lambda^2 x e^{-\lambda x}$

$\mu_{I_3}(x) = \mu_{I_2}(x) * \mu_{I_1}(x) = \dfrac{\lambda^3 x^2 e^{-\lambda x}}{2!}$

$\mu_{I_n}(x) = \dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}$

$\text{MAX } \mu_{I_n}(x) = \text{MAX}\left[\dfrac{\lambda^n x^{n-1} e^{-\lambda x}}{(n-1)!}\right]$

$x = \dfrac{n-1}{\lambda}$ //

~~J_1 : $\mu_{J_1}(x) = a(1-a)^{x-1}$, $x \geq 1$~~

~~J_2 : $\mu_{J_2}(x) = (x-1) a^2 (1-a)^{x-2}$; $1/a < x < 1 + 1/a$~~

~~J_n : $\mu_{J_n}(x) = \binom{x-1}{n-1} a^n (1-a)^{x-n}$; $\frac{n-1}{a} < x < 1 + \frac{n-1}{a}$~~

~~$J_1, J_2 \ldots J_n$ called~~