ANNA ADARSH COLLEGE FOR WOMEN (AUTONOMOUS), CHENNAI – 600040
END SEMESTER EXAMINATIONS-APRIL – 2026

Programme: (Common to B.Sc. Chemistry, Computer Science, Computer Science with Data Science, Computer Science with Artificial Intelligence and BCA)		**Batch:** 2025-2028		**Semester: II**
Course Title: Mathematics II		**Course Code:** 24UBSCH3E2A /24UBSC3E2A / 24UBSDS3E2A / 24UBSAI3E2A / / 24UBCAP3E2A		
Duration: 3 Hrs		**Maximum Marks:** 75		
Question No	**Question**	**Mark**	**K Level (K1 – K6)**	**(CO) (CO1-CO5)**
SECTION – A (10 X 2 = 20 Marks) **Answer Any TEN Questions**				
1	**State Bernoulli's Formula.** If u and v are functions of x, then: $\int u dv = uv - u'v_1 + u''v_2 - u'''v_3 + \dots$ Where: • $u', u'', u''', \dots$ are successive derivatives of u. • $v_1, v_2, v_3, \dots$ are successive integrals of v.			
2	**Compute** $\int x^3 e^{2x} \; dx$ $u' = 3x^2$, $u'' = 6x$, $u''' = 6$, $u'''' = 0$ $v = \int e^{2x} dx = \frac{e^{2x}}{2}$ $v_1 = \int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4}$ $v_2 = \int \frac{e^{2x}}{4} dx = \frac{e^{2x}}{8}$ $v_3 = \int \frac{e^{2x}}{8} dx = \frac{e^{2x}}{16}$			

	$\int x^3e^{2x}dx = (x^3)(\frac{e^{2x}}{2}) - (3x^2)(\frac{e^{2x}}{4}) + (6x)(\frac{e^{2x}}{8}) - (6)(\frac{e^{2x}}{16}) + C$ $\int x^3e^{2x}dx = \frac{e^{2x}}{8}(4x^3 - 6x^2 + 6x - 3) + C$
3	**Compute** $\int_0^{\frac{\pi}{2}} \sin^8 x \ dx$ $I_n = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$ $I_8 = \frac{8-1}{8} \cdot \frac{8-3}{8-2} \cdot \frac{8-5}{8-4} \cdot \frac{8-7}{8-6} \cdot \frac{\pi}{2}$ $I_8 = \frac{35\pi}{256}$
4	Define Odd and Even functions. An **even function** satisfies $f(-x) = f(x)$, showing symmetry about the y-axis; examples include $f(x) = x^2$ and $f(x) = \|x\|$. An **odd function** satisfies $f(-x) = -f(x)$, showing symmetry about the origin; examples include $f(x) = x^3$ and $f(x) = \tan(x)$.
5	**Find** a_0 **for** $f(x) = k, 0 \le x \le 2\pi$ The formula is $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$. Substituting $f(x) = k$, we get $a_0 = \frac{1}{\pi}\int_0^{2\pi} kdx$. Integrating k yields $\frac{k}{\pi}[x]_0^{2\pi} = \frac{k}{\pi}(2\pi - 0)$. Simplifying the expression, $a_0 = 2k$.
6	**Write the general form of 2nd Order differential equation.** The general form of a second-order ordinary differential equation (ODE) is $a(x)\frac{d^2y}{dx^2} + b(x)\frac{dy}{dx} + c(x)y = f(x)$. Alternatively, it is expressed as $y'' + P(x)y' + Q(x)y = R(x)$. If $f(x) = 0$, the equation is called homogeneous; if $f(x) \ne 0$, it is non-homogeneous.

7	**Eliminate the arbitrary constants a, b from** $z = (x+a)^2 + (y+b)^2 + c^2$ **to get a partial differential equation** $z = (x+a)^2 + (y+b)^2 + c^2$ $p = \frac{\partial z}{\partial x} = 2(x+a) \Longrightarrow (x+a) = \frac{p}{2}$ $q = \frac{\partial z}{\partial y} = 2(y+b) \Longrightarrow (y+b) = \frac{q}{2}$ $z = (\frac{p}{2})^2 + (\frac{q}{2})^2 + c^2$ $z - c^2 = \frac{p^2}{4} + \frac{q^2}{4}$ $4(z - c^2) = p^2 + q^2$
8	**Define Laplace transform.** The Laplace transform is an integral operator that converts a function of time, $f(t)$, into a complex frequency domain function, $F(s)$, defined $L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$ $s = \sigma + i\omega$ $t \geq 0$
9	**Find** $L(sin2t)$. $L\{\sin(at)\} = \frac{a}{s^2 + a^2}$ $L\{\sin(2t)\} = \frac{2}{s^2 + 2^2}$ $L\{\sin(2t)\} = \frac{2}{s^2 + 4}$

10	**Show that $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is a solenoidal** $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ $\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(y)$ $\nabla \cdot \vec{F} = 0 + 0 + 0$ $\nabla \cdot \vec{F} = 0$
11	**Define Curl of a vector.** $\text{curl } \vec{F} = \nabla \times \vec{F}$ $\nabla \times \vec{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$ $\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \mathbf{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) \mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \mathbf{k}$
12	**Define directional derivative of a vector.** $\nabla\phi \cdot \hat{n}$ $\nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k}$ $\hat{n} = \frac{\bar{a}}{\lvert\bar{a}\rvert}$
SECTION – B (5 X 5 = 25 Marks) **Answer Any FIVE Questions**	
13	**Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^4 x \; dx$** $\int_0^{\pi/2} \sin^7 x \cos^4 x \, dx$ $m = 7, n = 4$ $\frac{(m-1)(m-3)(m-5)\cdot(n-1)(n-3)}{(m+n)(m+n-2)(m+n-4)(m+n-6)(m+n-8)(m+n-10)}$ $\frac{(6\cdot4\cdot2)\cdot(3\cdot1)}{11\cdot9\cdot7\cdot5\cdot3\cdot1}$ $\frac{144}{10395}$ $\frac{16}{1155}$

14	**Derive the formula for $\int_0^{\frac{\pi}{2}} \sin^m x \, dx$** $I_m = \int_0^{\pi/2} \sin^m x \, dx$ $I_m = \int_0^{\pi/2} \sin^{m-1} x \cdot \sin x \, dx$ $u = \sin^{m-1} x \Longrightarrow du = (m-1)\sin^{m-2} x \cos x \, dx$ $dv = \sin x \, dx \Longrightarrow v = -\cos x$ $I_m = [-\sin^{m-1} x \cos x]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)(m-1)\sin^{m-2} x \cos x \, dx$ $I_m = 0 + (m-1)\int_0^{\pi/2} \sin^{m-2} x \cos^2 x \, dx$ $I_m = (m-1)\int_0^{\pi/2} \sin^{m-2} x (1 - \sin^2 x) \, dx$ $I_m = (m-1)\int_0^{\pi/2} \sin^{m-2} x \, dx - (m-1)\int_0^{\pi/2} \sin^m x \, dx$ $I_m = (m-1) I_{m-2} - (m-1) I_m$ $I_m + (m-1) I_m = (m-1) I_{m-2}$ $m I_m = (m-1) I_{m-2}$ $I_m = \frac{m-1}{m} I_{m-2}$ If m is odd: $I_m = \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdot \ldots \cdot \frac{2}{3} \cdot 1$ If m is even: $I_m = \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdot \ldots \cdot \frac{1}{2} \cdot \frac{\pi}{2}$
15	**Find the Fourier Series for the function $f(x) = x^2$ in $-\pi < x < \pi$** $f(x) = x^2, -\pi < x < \pi$ $f(-x) = (-x)^2 = x^2 \Longrightarrow f(x)$ is an even function $b_n = 0$ $a_0 = \frac{2}{\pi}\int_0^{\pi} x^2 \, dx = \frac{2}{\pi}[\frac{x^3}{3}]_0^{\pi} = \frac{2\pi^2}{3}$ $a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(nx) \, dx$ $u = x^2, dv = \cos(nx)dx$ $a_n = \frac{2}{\pi}[x^2(\frac{\sin(nx)}{n}) - 2x(-\frac{\cos(nx)}{n^2}) + 2(-\frac{\sin(nx)}{n^3})]_0^{\pi}$ $a_n = \frac{2}{\pi}[0 + \frac{2\pi\cos(n\pi)}{n^2} - 0] = \frac{4(-1)^n}{n^2}$

	$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$ $$x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos(nx)$$ $$x^2 = \frac{\pi^2}{3} - 4[\cos(x) - \frac{\cos(2x)}{4} + \frac{\cos(3x)}{9} - \dots]$$
16	**Solve $(D^2 + 16)y = e^{-3x} + cos4x$** $$(D^2 + 16)y = e^{-3x} + \cos(4x)$$ Find the Complementary Function (C.F.) by solving the homogeneous equation: $$(D^2 + 16)y = 0$$ Auxiliary equation: $m^2 + 16 = 0$ $$m^2 = -16$$ $$m = \pm\sqrt{-16} = \pm 4i$$ $$m = 0 \pm 4i$$ C.F. $y_c = e^{0x}(C_1 \cos(4x) + C_2 \sin(4x))$ $$y_c = C_1 \cos(4x) + C_2 \sin(4x)$$ Find the Particular Integral (P.I.) for e^{-3x}: P.I.$_1 = \frac{1}{D^2 + 16} e^{-3x}$ Substitute $D = -3$: P.I.$_1 = \frac{1}{(-3)^2 + 16} e^{-3x} = \frac{1}{9 + 16} e^{-3x} = \frac{1}{25} e^{-3x}$ Find the Particular Integral (P.I.) for $\cos(4x)$: P.I.$_2 = \frac{1}{D^2 + 16} \cos(4x)$ Substitute $D^2 = -(4^2) = -16$, which results in a zero denominator (case of failure). P.I.$_2 = x \frac{1}{\frac{d}{dD}(D^2 + 16)} \cos(4x)$ P.I.$_2 = x \frac{1}{2D} \cos(4x)$

	$\text{P.I.}_2 = \frac{x}{2}\frac{1}{D}\cos(4x)$ $\frac{1}{D}\cos(4x)$ means $\int \cos(4x)dx$: $\text{P.I.}_2 = \frac{x}{2}(\frac{\sin(4x)}{4})$ $\text{P.I.}_2 = \frac{x\sin(4x)}{8}$ General Solution $y = y_c + \text{P.I.}_1 + \text{P.I.}_2$: $$y = C_1\cos(4x) + C_2\sin(4x) + \frac{1}{25}e^{-3x} + \frac{x\sin(4x)}{8}$$
17	**Evaluate** $\mathbf{L(t^3 - e^{-2t} + \cos t - \sin 2t)}$ $$L\{t^3 - e^{-2t} + \cos(t) - \sin(2t)\}$$ $$L\{t^3\} - L\{e^{-2t}\} + L\{\cos(t)\} - L\{\sin(2t)\}$$ $$L\{t^n\} = \frac{n!}{s^{n+1}} \implies L\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$$ $$L\{e^{at}\} = \frac{1}{s-a} \implies L\{e^{-2t}\} = \frac{1}{s+2}$$ $$L\{\cos(at)\} = \frac{s}{s^2+a^2} \implies L\{\cos(t)\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$$ $$L\{\sin(at)\} = \frac{a}{s^2+a^2} \implies L\{\sin(2t)\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$ $$\frac{6}{s^4} - \frac{1}{s+2} + \frac{s}{s^2+1} - \frac{2}{s^2+4}$$
18	**If** $\nabla\emptyset = 2xyz^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$**. Find** $\emptyset(-1,2,2) = 4$ $\nabla\phi = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ $$\frac{\partial\phi}{\partial x} = 2xyz^3$$ $$\frac{\partial\phi}{\partial y} = x^2z^3$$ $$\frac{\partial\phi}{\partial z} = 3x^2yz^2$$

	Integrate the first equation with respect to x: $$\phi(x,y,z) = \int 2xyz^3\,dx = x^2yz^3 + C_1(y,z)$$ Differentiate this ϕ with respect to y and compare with $\frac{\partial \phi}{\partial y}$: $$\frac{\partial \phi}{\partial y} = x^2z^3 + \frac{\partial C_1}{\partial y}$$ $$x^2z^3 = x^2z^3 + \frac{\partial C_1}{\partial y}$$ $$\frac{\partial C_1}{\partial y} = 0 \Longrightarrow C_1(y,z) = C_2(z)$$ So, $\phi(x,y,z) = x^2yz^3 + C_2(z)$ Differentiate this ϕ with respect to z and compare with $\frac{\partial \phi}{\partial z}$: $$\frac{\partial \phi}{\partial z} = 3x^2yz^2 + \frac{dC_2}{dz}$$ $$3x^2yz^2 = 3x^2yz^2 + \frac{dC_2}{dz}$$ $\frac{dC_2}{dz} = 0 \Longrightarrow C_2(z) = C$ (a constant) The potential function is $\phi(x,y,z) = x^2yz^3 + C$ Use the initial condition $\phi(-1,2,2) = 4$ to find C: $$4 = (-1)^2(2)(2)^3 + C$$ $$4 = (1)(2)(8) + C$$ $$4 = 16 + C$$ $$C = 4 - 16 = -12$$ The final function ϕ is: $$\phi(x,y,z) = x^2yz^3 - 12$$
19	Show that $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ $$\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$ $$r = \sqrt{x^2+y^2+z^2}$$ $$\frac{\partial r}{\partial x} = \frac{x}{r},\ \frac{\partial r}{\partial y} = \frac{y}{r},\ \frac{\partial r}{\partial z} = \frac{z}{r}$$

	$\nabla\left(\frac{1}{r}\right) = \mathbf{i}\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + \mathbf{j}\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + \mathbf{k}\frac{\partial}{\partial z}\left(\frac{1}{r}\right)$ $\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{\partial r}{\partial x} = -\frac{1}{r^2}\left(\frac{x}{r}\right) = -\frac{x}{r^3}$ $\nabla\left(\frac{1}{r}\right) = -\frac{x}{r^3}\mathbf{i} - \frac{y}{r^3}\mathbf{j} - \frac{z}{r^3}\mathbf{k}$ $\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^3}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$ $\nabla\left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$
SECTION – C (3X 10 = 30 Marks) **Answer Any THREE Questions**	
20	**Derive the reduction formula for** $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \ dx$ $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x dx$ $I_{m,n} = \int_0^{\pi/2} \sin^{m-1} x(\sin x \cos^n x)dx$ Let $u = \sin^{m-1} x \Longrightarrow du = (m-1)\sin^{m-2} x \cos x dx$ Let $dv = \cos^n x \sin x dx \Longrightarrow v = -\frac{\cos^{n+1} x}{n+1}$ $I_{m,n} = [-\frac{\sin^{m-1} x \cos^{n+1} x}{n+1}]_0^{\pi/2} + \frac{m-1}{n+1}\int_0^{\pi/2} \cos^{n+1} x(\sin^{m-2} x \cos x)dx$ $I_{m,n} = 0 + \frac{m-1}{n+1}\int_0^{\pi/2} \sin^{m-2} x \cos^{n+2} x dx$ $I_{m,n} = \frac{m-1}{n+1}\int_0^{\pi/2} \sin^{m-2} x \cos^n x(1 - \sin^2 x)dx$ $I_{m,n} = \frac{m-1}{n+1}\left[\int_0^{\pi/2} \sin^{m-2} x \cos^n x dx - \int_0^{\pi/2} \sin^m x \cos^n x dx\right]$ $I_{m,n} = \frac{m-1}{n+1} I_{m-2,n} - \frac{m-1}{n+1} I_{m,n}$

	$I_{m,n}(1+\frac{m-1}{n+1}) = \frac{m-1}{n+1}\ I_{m-2,n}$ $I_{m,n}(\frac{n+1+m-1}{n+1}) = \frac{m-1}{n+1}\ I_{m-2,n}$ $I_{m,n}(\frac{m+n}{n+1}) = \frac{m-1}{n+1}\ I_{m-2,n}$ $\mathbf{I_{m,n} = \frac{m-1}{m+n}\ I_{m-2,n}}$
21	**Find the Fourier Series for the function $f(x) = x(2\pi - x)$ in $(0,2\pi)$.** $f(x) = x(2\pi - x) = 2\pi x - x^2$ in $(0, 2\pi)$ $a_0 = \frac{1}{\pi}\int_0^{2\pi}(2\pi x - x^2)dx = \frac{1}{\pi}\ [\pi x^2 - \frac{x^3}{3}\ ]_0^{2\pi} = \frac{1}{\pi}\ (4\pi^3 - \frac{8\pi^3}{3}) = \frac{4\pi^2}{3}$ $a_n = \frac{1}{\pi}\int_0^{2\pi}(2\pi x - x^2)\cos(nx)dx$ $u = 2\pi x - x^2 \Longrightarrow u' = 2\pi - 2x \Longrightarrow u'' = -2 \Longrightarrow u''' = 0$ $v = \cos(nx) \Longrightarrow v_1 = \frac{\sin(nx)}{n} \Longrightarrow v_2 = -\frac{\cos(nx)}{n^2} \Longrightarrow v_3 = -\frac{\sin(nx)}{n^3}$ $a_n = \frac{1}{\pi}\ [(2\pi x - x^2)\ \frac{\sin(nx)}{n} - (2\pi - 2x)(-\frac{\cos(nx)}{n^2}) + (-2)(-\frac{\sin(nx)}{n^3})]_0^{2\pi}$ $a_n = \frac{1}{\pi}\ [0 + \frac{2\pi - 2x}{n^2}\cos(nx) + \frac{2\sin(nx)}{n^3}\ ]_0^{2\pi}$ $a_n = \frac{1}{\pi}\ [(\frac{-2\pi}{n^2}\cos(2n\pi) + 0) - (\frac{2\pi}{n^2}\cos(0) + 0)] = \frac{1}{\pi}\ [-\frac{2\pi}{n^2} - \frac{2\pi}{n^2}\ ] = -\frac{4}{n^2}$ $b_n = \frac{1}{\pi}\ [(2\pi x - x^2)(-\frac{\cos(nx)}{n}) - (2\pi - 2x)(-\frac{\sin(nx)}{n^2}) + (-2)(\frac{\cos(nx)}{n^3})]_0^{2\pi}$ $b_n = \frac{1}{\pi}\ [-\frac{2\pi x - x^2}{n}\cos(nx) + \frac{2\pi - 2x}{n^2}\sin(nx) - \frac{2\cos(nx)}{n^3}\ ]_0^{2\pi}$ $b_n = \frac{1}{\pi}\ [(0 + 0 - \frac{2}{n^3}) - (0 + 0 - \frac{2}{n^3})] = 0$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx)$ $\mathbf{f(x) = \frac{2\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{\cos(nx)}{n^2}}$

22	**Solve** $(2D^2 + 3D + 5)y = e^x cosx$ For y_c, the auxiliary equation is $2m^2 + 3m + 5 = 0$: $$m = \frac{-3 \pm \sqrt{9-4(2)(5)}}{2(2)} = \frac{-3 \pm \sqrt{-31}}{4} = -\frac{3}{4} \pm i\,\frac{\sqrt{31}}{4}$$ $$y_c = e^{-\frac{3}{4}x}\left(C_1 \cos\frac{\sqrt{31}}{4}x + C_2 \sin\frac{\sqrt{31}}{4}x\right)$$ For y_p: $$y_p = \frac{1}{2D^2+3D+5}e^x \cos x = e^x \frac{1}{2(D+1)^2+3(D+1)+5}\cos x$$ $$y_p = e^x \frac{1}{2(D^2+2D+1)+3D+3+5}\cos x$$ $$y_p = e^x \frac{1}{2D^2+7D+10}\cos x$$ Substitute $D^2 = -1^2 = -1$: $$y_p = e^x \frac{1}{2(-1)+7D+10}\cos x = e^x \frac{1}{7D+8}\cos x$$ Rationalize the operator: $$y_p = e^x \frac{7D-8}{(7D+8)(7D-8)}\cos x = e^x \frac{7D-8}{49D^2-64}\cos x$$ Substitute $D^2 = -1$: $$y_p = e^x \frac{7D-8}{49(-1)-64}\cos x = e^x \frac{7(-\sin x) - 8\cos x}{-113}$$ $$y_p = \frac{e^x}{113}(7\sin x + 8\cos x)$$ $$\mathbf{y = e^{-\frac{3}{4}x}\left(C_1 \cos\frac{\sqrt{31}}{4}x + C_2 \sin\frac{\sqrt{31}}{4}x\right) + \frac{e^x}{113}(7\sin x + 8\cos x)}$$
23	Find $\boldsymbol{L^{-1}}\left[\frac{\boldsymbol{7s-1}}{\boldsymbol{(s+1)(s+2)(s+3)}}\right]$ Use partial fraction decomposition: $$\frac{7s-1}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

	$$7s - 1 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$ Set $s = -1$: $$7(-1) - 1 = A(-1+2)(-1+3) \Longrightarrow -8 = A(1)(2) \Longrightarrow A = -4$$ Set $s = -2$: $$7(-2) - 1 = B(-2+1)(-2+3) \Longrightarrow -15 = B(-1)(1) \Longrightarrow B = 15$$ Set $s = -3$: $$7(-3) - 1 = C(-3+1)(-3+2) \Longrightarrow -22 = C(-2)(-1) \Longrightarrow -22 = 2C \Longrightarrow C = -11$$ Substitute A, B, C back into the expression: $$\frac{7s-1}{(s+1)(s+2)(s+3)} = \frac{-4}{s+1} + \frac{15}{s+2} + \frac{-11}{s+3}$$ Apply the inverse Laplace transform, using the linearity property and $L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$: $$L^{-1}\left[\frac{-4}{s+1}\right] + L^{-1}\left[\frac{15}{s+2}\right] + L^{-1}\left[\frac{-11}{s+3}\right]$$ $$= -4L^{-1}\left[\frac{1}{s+1}\right] + 15L^{-1}\left[\frac{1}{s+2}\right] - 11L^{-1}\left[\frac{1}{s+3}\right]$$ $$= -4e^{-t} + 15e^{-2t} - 11e^{-3t}$$ $$\mathbf{L^{-1}\left[\frac{7s-1}{(s+1)(s+2)(s+3)}\right] = -4e^{-t} + 15e^{-2t} - 11e^{-3t}}$$
24	Determine the value of 'a' such that $\boldsymbol{a})(\mathbf{3x - 2y + z})\vec{\boldsymbol{i}} + (\boldsymbol{4x + ay - z})\vec{\boldsymbol{j}} + (\boldsymbol{x - y + 2z})\vec{\boldsymbol{k}}$ is solenoidal b) $\vec{\boldsymbol{F}} = (\boldsymbol{axy - z^2})\vec{\boldsymbol{i}} + (\boldsymbol{x^2 + 2yz})\vec{\boldsymbol{j}} + (\boldsymbol{y^2 - axz})\vec{\boldsymbol{k}}$ is irrotational a) A vector field $\vec{V} = P\hat{i} + Q\hat{j} + R\hat{k}$ is solenoidal if its divergence is zero, i.e., $\nabla \cdot \vec{V} = 0$. $$\vec{V} = (3x - 2y + z)\hat{i} + (4x + ay - z)\hat{j} + (x - y + 2z)\hat{k}$$ $$P = 3x - 2y + z$$ $$Q = 4x + ay - z$$ $$R = x - y + 2z$$ $$\nabla \cdot \vec{V} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\frac{\partial P}{\partial x} = 3$$

$$\frac{\partial Q}{\partial y} = a$$

$$\frac{\partial R}{\partial z} = 2$$

$$\nabla \cdot \vec{V} = 3 + a + 2 = 5 + a$$

For the field to be solenoidal, $5 + a = 0$

$$a = -5$$

b) A vector field $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ is irrotational if its curl is zero, i.e., $\nabla \times \vec{F} = \vec{0}$.

$$\vec{F} = (axy - z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 - axz)\hat{k}$$

$$P = axy - z^2$$

$$Q = x^2 + 2yz$$

$$R = y^2 - axz$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$\nabla \times \vec{F} = \hat{i}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) - \hat{j}\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + \hat{k}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$\frac{\partial R}{\partial y} = 2y$$

$$\frac{\partial Q}{\partial z} = 2y$$

$$\frac{\partial R}{\partial x} = -az$$

$$\frac{\partial P}{\partial z} = -2z$$

	$$\frac{\partial Q}{\partial x} = 2x$$ $$\frac{\partial P}{\partial y} = ax$$ $$\nabla \times \vec{F} = \hat{\imath}(2y - 2y) - \hat{\jmath}(-az - (-2z)) + \hat{k}(2x - ax)$$ $$\nabla \times \vec{F} = \hat{\imath}(0) - \hat{\jmath}(2z - az) + \hat{k}(2x - ax)$$ $$\nabla \times \vec{F} = (az - 2z)\hat{\jmath} + (2x - ax)\hat{k}$$ For the field to be irrotational, $\nabla \times \vec{F} = \vec{0}$. This requires the coefficients of $\hat{\jmath}$ and $\hat{k}$ to be zero: $$az - 2z = 0 \Longrightarrow z(a - 2) = 0 \Longrightarrow a - 2 = 0 \Longrightarrow a = 2$$ $$2x - ax = 0 \Longrightarrow x(2 - a) = 0 \Longrightarrow 2 - a = 0 \Longrightarrow a = 2$$ Both conditions give the same value for a. $$a = 2$$