Answer key : STATISTICS - II

Course Code : 24UBCAP3E2B

Section - A

1. $P(A \cup B) = P(A) + P(B)$, when A & B are called mutually exclusive events.

2. i) $P(A) \geq 0$, $\forall A$

   ii) $P(S) = 1$

   iii) $P(A \cup B) = P(A) + P(B)$, are called axioms on prob.

3. $P(53 \text{ Sunday}) = \frac{2}{7}$

4. $P(X < 3) = P(X = -2) + P(X = 0)$

   $= 0.2 + 0.25 = 0.45$

5. Every outcome of a random experiment is classified in to success or failure called Bernoullian trial

   (or) $p + q = 1$

6. $\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \frac{ax^3}{3}\Big|_0^1 = 1$

   $\therefore \boxed{a = 3}$

7. $n \to \infty$

   $p \to 0$ Binomial $\Rightarrow$ poisson

8. α - level of significance, which is how much error is acceptable

9. Mean = 0 & variance $= \frac{n}{n-2}$, $n > 2$

10. Goodness of fit, Independent of Attributes

11. Rejecting H_0, when H_0 is true is called Type-I error.

$P_r\{\text{Type-I error}\} = \alpha$

12. Statement about population parameter is called hypothesis.

Section - B

13. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

i) $P(\text{Exactly two heads}) = \frac{3}{8}$

ii) $P(\text{At least one Head}) = 1 - P_r\{\text{no Head}\}$

$= 1 - \frac{1}{8} = 7/8$

iii) $P\{\text{At most one head}\} = \frac{4}{8} = 1/2$

14. Given Mean = 3 $\Rightarrow$ $\boxed{\lambda = 3}$

$P(X = x) = \frac{e^{-3} 3^x}{x!}$, $x = 0, 1, 2.$

X - Number of Accident.

(i) No accident in a year = $N\,p(x=0)$

$= 1000 \times \frac{e^{-3} \cdot 3^0}{0!} = e^{-3} \times 1000$

$= 49.79$

$\simeq 50$

(ii) P(more than 1 Accidents) $= p(x>1)$

$= 1 - p(x \leq 1)$

$= 1 - \{p(x=0) + p(x=1)\}$

$= 1 - \{e^{-3} + 3e^{-3}\}$

$= 1 - 4e^{-3}$

$= 0.8$

more than 1 accident $= N \times P(x>1)$

$= 1000 \times 0.8$

$= 800$

15. Normal distribution Properties

Standards that to waste at least 5.

Given, $\mu = 60$, $\sigma = 5$

when $x = 45$, $Z = \frac{45-60}{5} = -3$

$x = 56$, $Z = \frac{56-60}{5} = -0.8$

$x = 65$, $Z = \frac{65-60}{5} = 1$

(i) $P\{45 < x < 65\} = P(-3 < z < 1)$

$= P(0 < z < 3) + P(0 < z < 1)$

$= 0.83995$

ii) $P(x < 56) = P(z < -0.8)$

$= 0.5 - P(-0.8 < z < 0)$

$= 0.5 - 0.2881 = 0.2119$

12 $n = 10$, $\mu = 100$, $\bar{x} = 97.2$

$s = 13.54$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \sim t_{(n-1)\,d.f}$

$t = -0.62$

$|t| = 0.62$

table value: 2.262

$\therefore$ cal $t <$ tab t

$\therefore$ Inference: accept H_0

(or) These data support the Assumption of the pop mean IQ.

13. Students has to write a test procedure of χ^2 - goodness of fit

15 $p_1 = 0.05$ & $q_1 = 0.95$

$p_2 = 0.033$ & $q_2 = 0.967$

$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{20 + 10}{400 + 300} = 0.04$

$q = 0.96$

$$z = \frac{0.05 - 0.032}{\sqrt{0.04 \times 0.96 \left(\frac{1}{200} + \frac{1}{300}\right)}}$$

$$= \frac{0.018}{0.0192} = 0.96$$

Table value = 1.645 at 5% level

$\therefore$ Cal z < Table z

Conclusion: we may accept / fail to reject hypothesis H_0

Thus there is no improvement

Section - C

20. $P(A) = 0.5$, $P(B) = 0.3$, $P(C) = 0.2$

Let Effect of car is tested

$P(G|A) = 0.05$, $P(G|B) = 0.03$

$P(G|A) = 0.97$

$$P(G) = P(A)P(G|A) + P(B)P(G|B) + P(C)P(G|C)$$

$$= 0.5 \times 0.95 + 0.3 \times 0.97 + 0.2 \times 0.97$$

$$= 0.948$$

21. P - Probability of defective

$p = 0.1$, $q = 0.9$

$n = 20$

$$P(x) = {}^{20}C_x (0.1)^x (0.9)^{20-x}, \quad x = 0, 1, 2, \ldots 20$$

i) $p(x=2) = {}^{20}C_2 (0.1)^2 (0.5)^{18}$

$= 0.2852$

ii) $p(x \leq 2) = p(x=0) + p(x=1) + p(x=2)$

$= 0.1216 + 0.27 + 0.2852$

$= 0.677$

iii) $p(x \geq 2) = 1 - p(x < 2)$

$= 1 - (0.1216 + 0.27)$

$= 1 - 0.3916 = 0.6084$

iv) $p(1 \leq x \leq 2) = 0.27 + 0.2852 + 0.1501$

$= 0.7053$

22. $f(x) = \theta e^{-\theta x}, \quad x > 0$

Mean $= \frac{1}{\theta}$ & var $= \frac{1}{\theta^2}$

23. Applying t-test

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2 df}$$

$\bar{x}_1 = 60, \quad \bar{x}_2 = 66 \quad ; \quad n_1 = 9$ & $n_2 = 11$

$s_1^2 = 21.1111, \quad s_2^2 = 30.3636$

$\therefore t = -2.28$

$|t| = 2.28$

Table value at 18 d.f is 2.101

$\therefore$ cal t > Tab t

we may Reject our Null hypothesis i.e Marks obtain by regular & part time students are difference.

24. One way classification.

S.V	d.f	S.S	MS	F-Ratio	F-table
Between rows	3-1	16.8	$\frac{16.8}{2} = 8.4$	$\frac{8.4}{7.47} = 1.13$	$F(2,9) = 4.26$
Error	12-3	67.2	$\frac{67.2}{9} = 7.47$		
Total	11	84			

$\therefore$ Cal F < Tab F

$\therefore$ we may Accept Null hypothesis H_0.

i.e There is no significant difference between the varieties.